Chronological Backtracking

A practical guide to chronological backtracking in SAT solvers

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Introduction

- Modern SAT solvers use CDCL
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- A lot of things can go wrong!
- We will see:
  - A reminder of CDCL
  - What is chronological backtracking?
  - What breaks?
  - How to fix it?
  - How to make it stronger?
  - What is next?

Framework and Notations

Definition

Let φ be a propositional formula in CNF, and $\pi = \tau \cup \omega$ be a conjunctive set of literals.

- π is a **partial assignment** of ϕ, also called the trail.
- τ^d is the set of decisions in π.
- τ is the set of **propagated literals**.
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(Trail construction) Any literal ℓ_i in the propagation queue ω is a consequence of some clause C in φ and $\tau \cup \omega[0 : i-1]$ or is a decision. That is,
$\forall \ell \in \omega.\ \exists C \in F.\ [(\tau \cup \omega[0 : i-1]) \wedge C \models \ell] \vee [\ell \in \tau^d]$.

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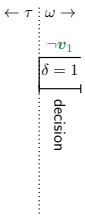

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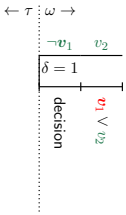

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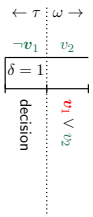

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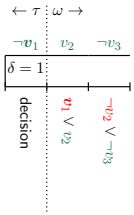

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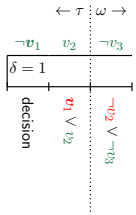

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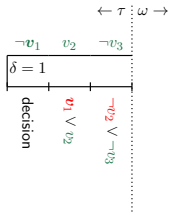

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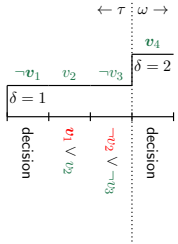

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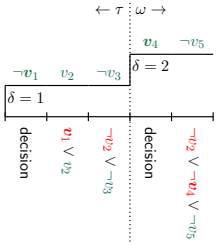

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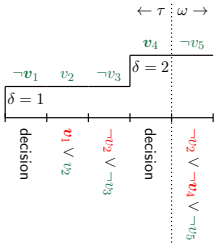

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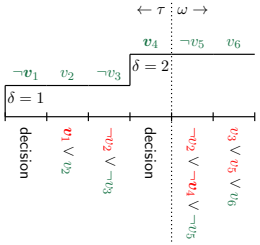

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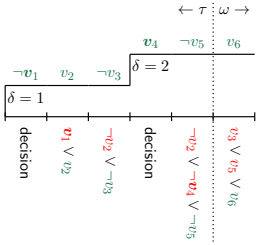

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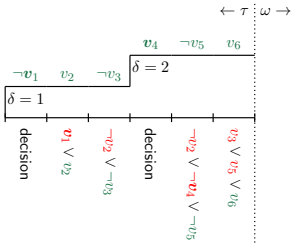

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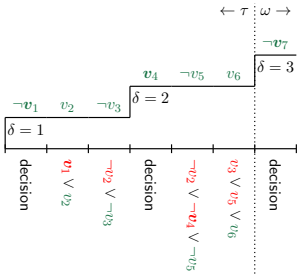

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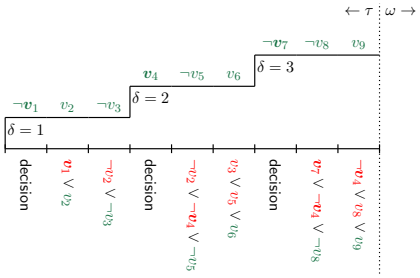

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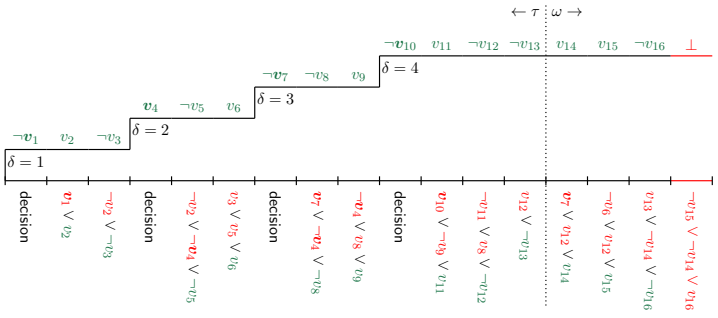

Conflict Driven Clause Learning

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
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C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
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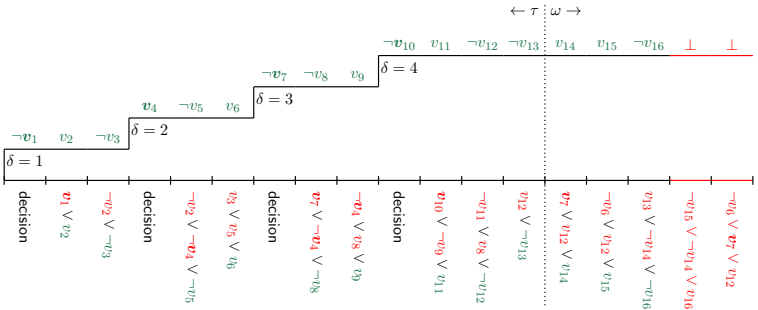

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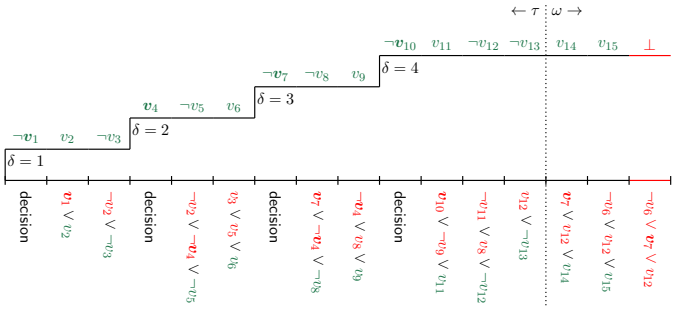

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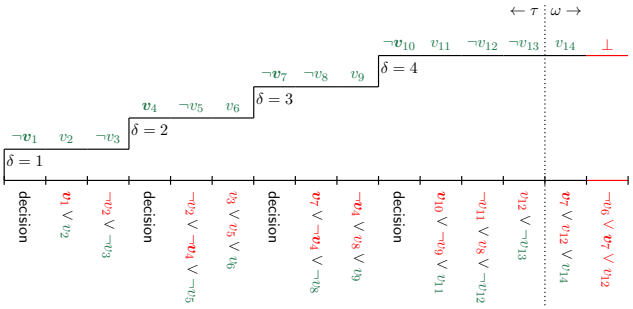

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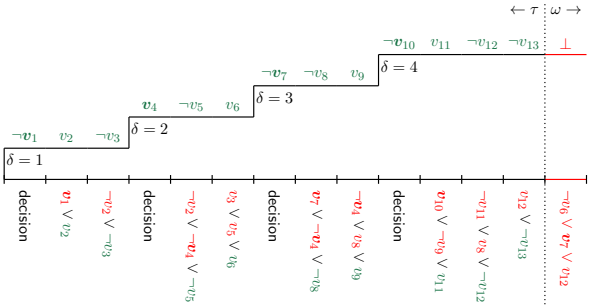

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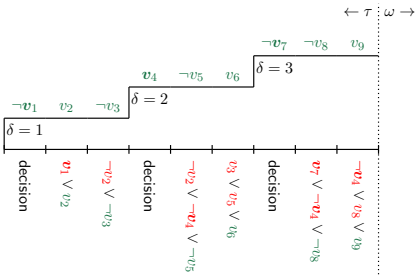

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\end{aligned}
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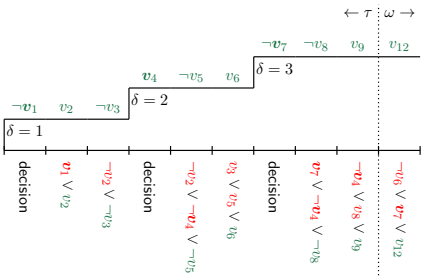

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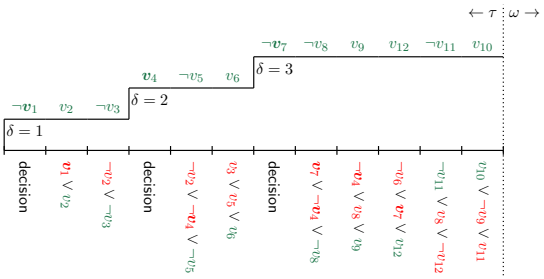

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\end{aligned}
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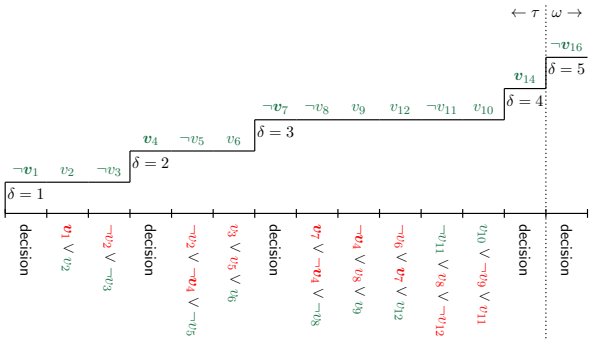

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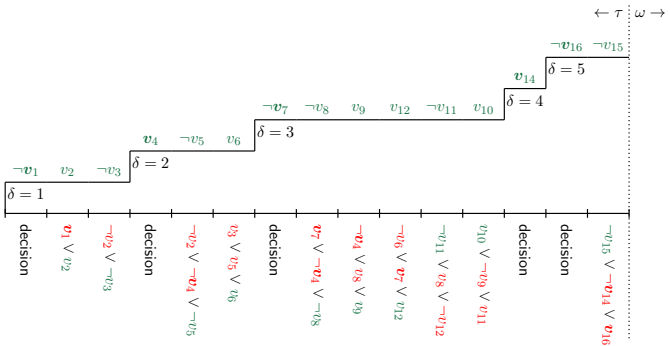


Backtracking Level

Different levels of backtracking

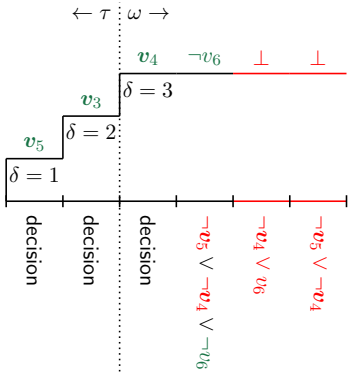
What if the there is a gap between the highest decision level and the second-highest decision level in the learned clause?

NCB. Backtrack literals as long as the learned clause is unit.

What if we backtrack at a different level? We can backtrack anywhere between the highest decision level minus one, and the second-highest decision level.

Non-Chronological Backtracking

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \vee v_3 \\
C_2 &= \underline{\neg v_2} \vee \underline{v_4} \\
C_3 &= \underline{\neg v_1} \vee \underline{v_4} \\
C_4 &= \underline{\neg v_5} \vee \underline{\neg v_4} \vee \underline{\neg v_6} \\
C_5 &= \underline{\neg v_4} \vee \underline{v_6} \\
C_6 &= \underline{\neg v_5} \vee \underline{\neg v_4}
\end{aligned}
$$



Non-Chronological Backtracking

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \vee v_3 \\
C_2 &= \underline{\neg v_2} \vee \underline{v_4} \\
C_3 &= \underline{\neg v_1} \vee \underline{v_4} \\
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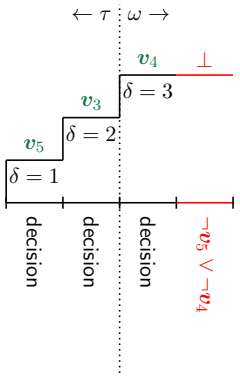
C_6 &= \underline{\neg v_5} \vee \underline{\neg v_4}
\end{aligned}
$$



Non-Chronological Backtracking

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\begin{aligned}
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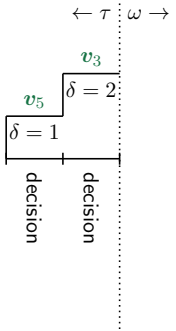
$$



Non-Chronological Backtracking

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\begin{aligned}
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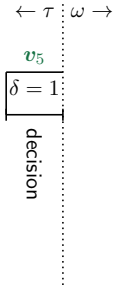

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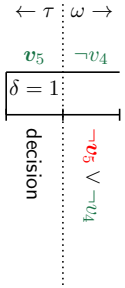

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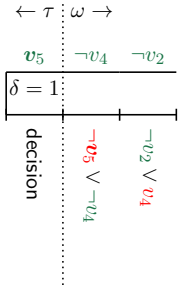
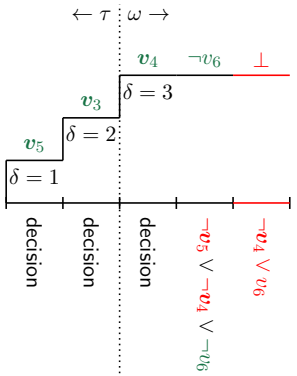
$$



Chronological Backtracking



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Chronological Backtracking

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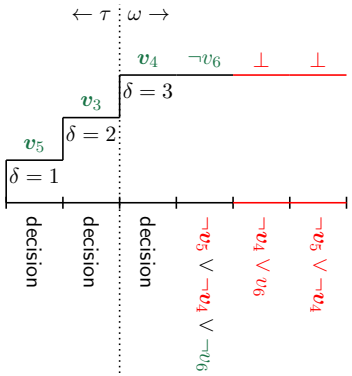

Chronological Backtracking

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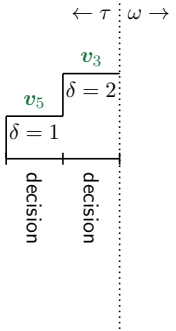

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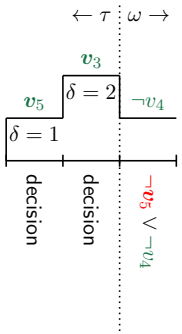
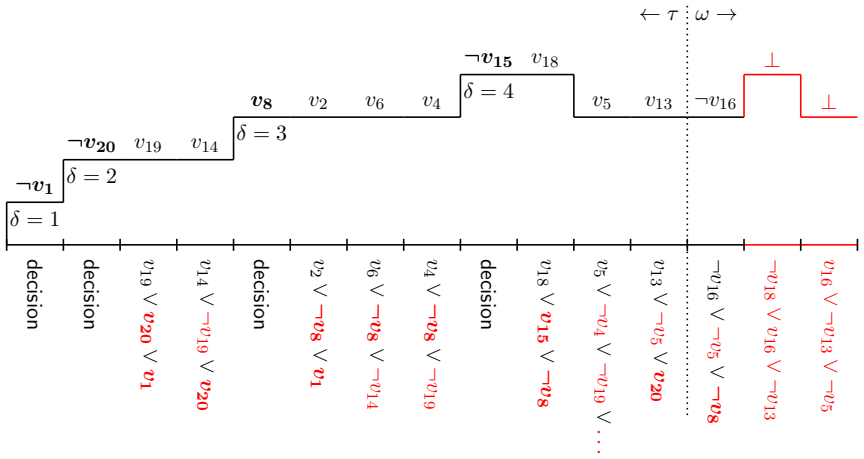


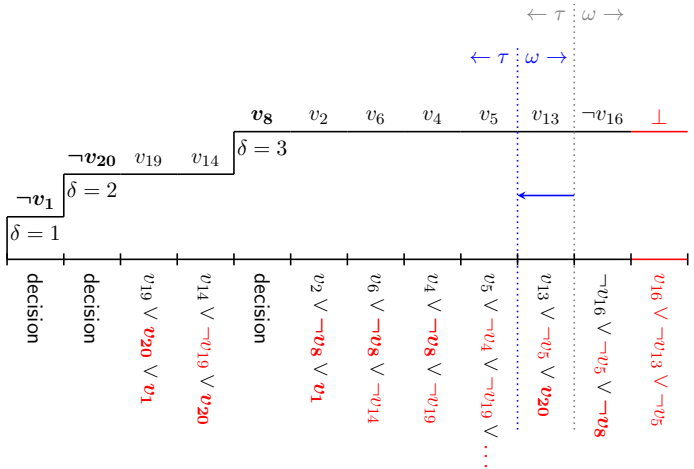
Weak Chronological Backtracking

Weak Chronological Backtracking is the minimum amount of changes that are necessary to have a functioning SAT solver. Without it, the solver becomes unsound.
We will discuss Strong Chronological Backtracking later.

Problem - Multi-Level Conflicts + Premature Propagation



Solution 1 - Multiple Backtracks



Solution 2 - Find Lowest

Solution 2

Do not stop at the first conflict. Return the lowest conflict clause found after complete propagation.

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Do not stop at the first conflict. Return the lowest conflict clause found after complete propagation.

Pros and Cons

Multiple backtracks	**Continue propagation**
Minimal changes	More changes
More conflicts	Less conflicts
Propagate multiple times	Propagates further

Solution 2 - Find Lowest

Solution 2

Do not stop at the first conflict. Return the lowest conflict clause found after complete propagation.

Pros and Cons

Multiple backtracks	Continue propagation
Minimal changes	More changes
More conflicts	Less conflicts
Propagate multiple times	Propagates further

Conclusion

It is not clear which solution is better. A simple approach to implement the first solution is to avoid adding the literal on the trail prematurely.

Feature - Watched Literals [Moskewicz et al., 2001]

Definition

Let C be a clause with at least two literals. C is watched by two literals ℓ_1 and ℓ_2 if $C = \ell_1 \vee \ell_2 \vee \ldots$.

Invariant

(Watched literals) Let $\pi = \tau \cup \omega$ be a partial assignment, and $C = \ell_1 \vee \ell_2 \vee \ldots$ be a clause watched by ℓ_1 and ℓ_2. If one of the watch literals is falsified by the trail τ, then the other must be satisfied by the partial assignment $\pi = \tau \cup \omega$.

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Feature - Watched Literals

Theorem

Let $C = \ell_1 \vee \ell_2 \vee \dots$ be a clause watched by ℓ_1 and ℓ_2. If Invariant "Watched literals" holds, then propagating any literal other than $\neg\ell_1$ and $\neg\ell_2$ cannot cause a conflict nor a unit propagation with C.

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Let $C = \ell_1 \vee \ell_2 \vee \dots$ be a clause watched by ℓ_1 and ℓ_2. If Invariant "Watched literals" holds, then propagating any literal other than $\neg\ell_1$ and $\neg\ell_2$ cannot cause a conflict nor a unit propagation with C.

Corollary

If Invariant "Watched literals" holds, then Invariant "Trail sanity" holds.

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Theorem

Let $C = \ell_1 \vee \ell_2 \vee \dots$ be a clause watched by ℓ_1 and ℓ_2. If Invariant "Watched literals" holds, then propagating any literal other than $\neg\ell_1$ and $\neg\ell_2$ cannot cause a conflict nor a unit propagation with C.

Corollary

If Invariant "Watched literals" holds, then Invariant "Trail sanity" holds.

Corollary

If a clause is conflicting, then both its watch literals are in the propagation queue ω.

Watched Literals Example

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\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \neg v_5 \\
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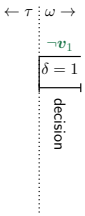

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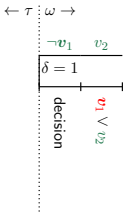

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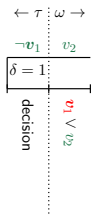

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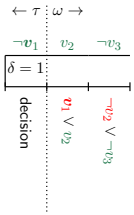

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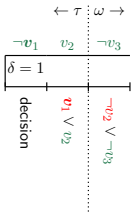

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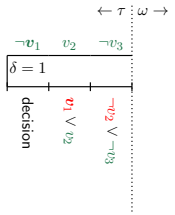

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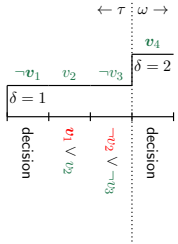

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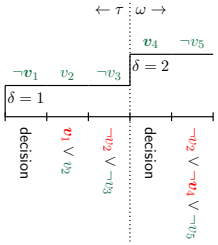

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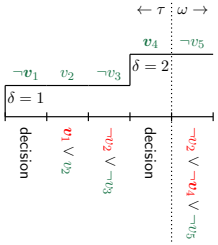

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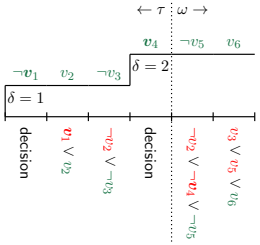

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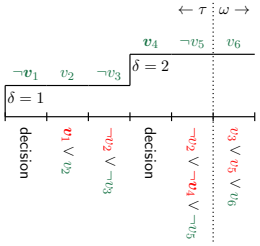

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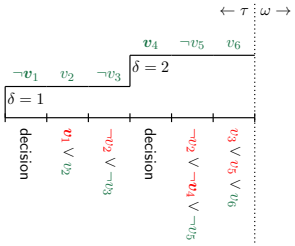

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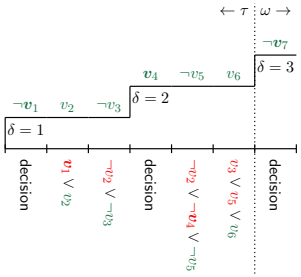

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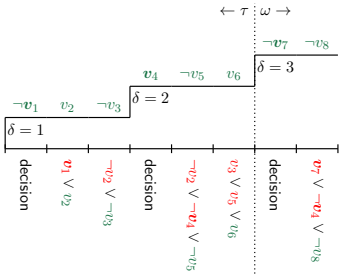
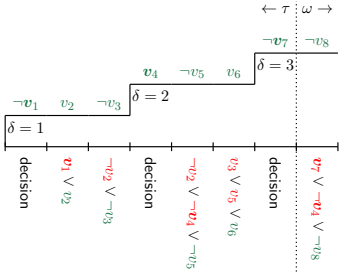
\end{aligned}
$$



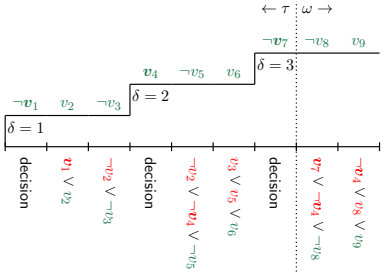
Watched Literals Example

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
C_4 &= v_3 \vee v_5 \vee v_6 \\
C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
C_6 &= \neg v_4 \vee v_8 \vee v_9 \\
C_7 &= v_{10} \vee \neg v_9 \vee v_{11} \\
C_8 &= \neg v_{11} \vee v_8 \vee \neg v_{12} \\
C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
C_{11} &= \neg v_6 \vee v_{12} \vee v_{15} \\
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



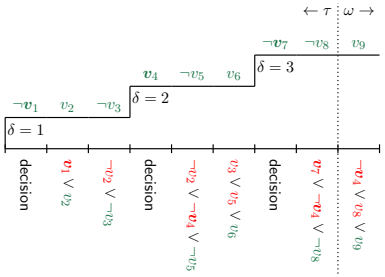
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \neg v_2 \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= v_3 \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \neg v_4 \vee \underline{\neg v_8} \\
C_6 &= \neg v_4 \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee v_{11} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \neg v_{12} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \neg v_{16} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



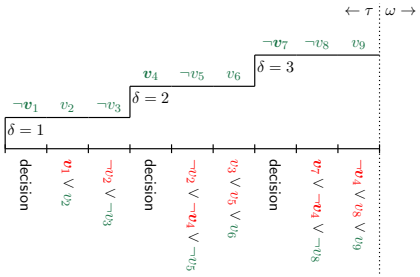
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \neg v_2 \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= v_3 \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \neg v_4 \vee \underline{\neg v_8} \\
C_6 &= \neg v_4 \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee v_{11} \\
C_8 &= \underline{\neg v_{11}} \vee v_8 \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \neg v_{16} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



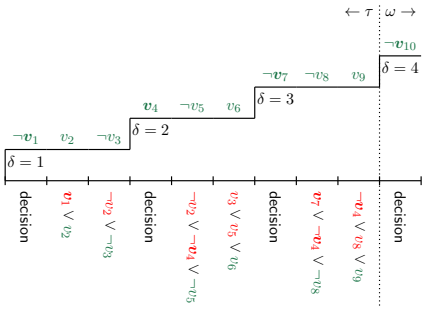
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \neg v_2 \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= v_3 \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \neg v_4 \vee \underline{\neg v_8} \\
C_6 &= \neg v_4 \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \neg v_9 \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee v_8 \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \neg v_{16} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



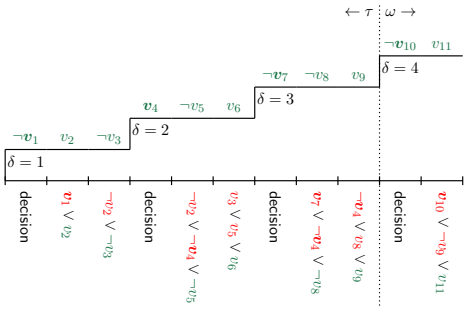
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \underline{\neg v_4} \vee \underline{\neg v_8} \\
C_6 &= \underline{\neg v_4} \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \underline{\neg v_{16}} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee \underline{v_{16}}
\end{aligned}
$$



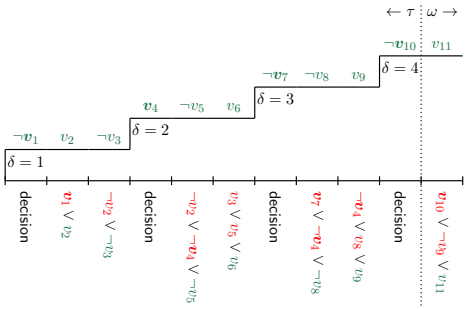
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \neg v_2 \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= v_3 \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \neg v_4 \vee \underline{\neg v_8} \\
C_6 &= \neg v_4 \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \neg v_9 \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee v_8 \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \underline{\neg v_{16}} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



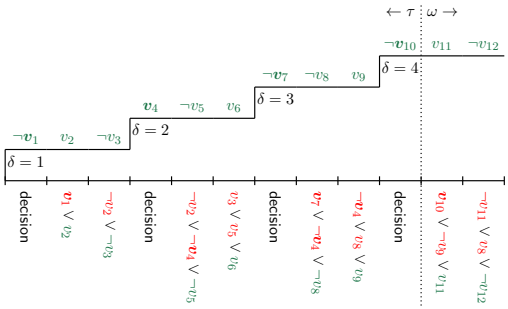
Watched Literals Example

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
C_4 &= v_3 \vee v_5 \vee v_6 \\
C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
C_6 &= \neg v_4 \vee v_8 \vee v_9 \\
C_7 &= v_{10} \vee \neg v_9 \vee v_{11} \\
C_8 &= \neg v_{11} \vee v_8 \vee \neg v_{12} \\
C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
C_{11} &= \neg v_6 \vee v_{12} \vee v_{15} \\
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



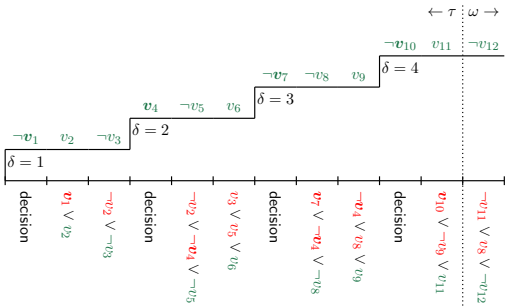
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \underline{\neg v_4} \vee \underline{\neg v_8} \\
C_6 &= \underline{\neg v_4} \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \underline{\neg v_{16}} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



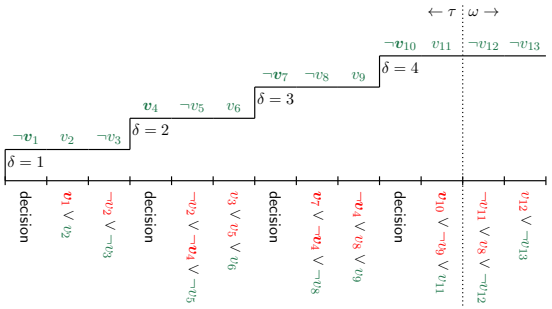
Watched Literals Example

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
C_4 &= v_3 \vee v_5 \vee v_6 \\
C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
C_6 &= \neg v_4 \vee v_8 \vee v_9 \\
C_7 &= v_{10} \vee \neg v_9 \vee v_{11} \\
C_8 &= \neg v_{11} \vee v_8 \vee \neg v_{12} \\
C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
C_{11} &= \neg v_6 \vee v_{12} \vee v_{15} \\
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



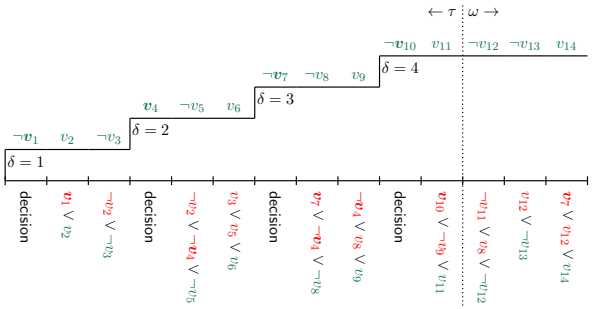
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \underline{\neg v_4} \vee \underline{\neg v_8} \\
C_6 &= \underline{\neg v_4} \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \underline{\neg v_{16}} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



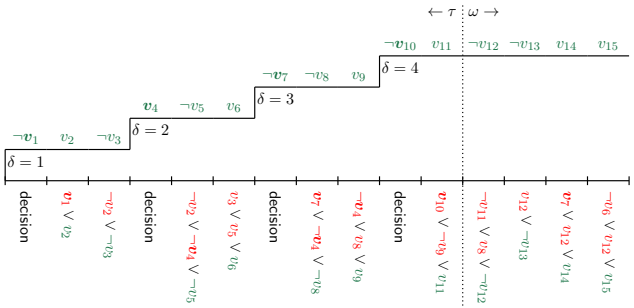
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \underline{\neg v_4} \vee \underline{\neg v_8} \\
C_6 &= \underline{\neg v_4} \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \neg v_{16} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



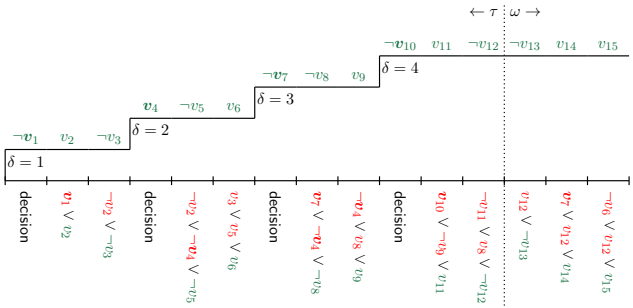
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \underline{\neg v_4} \vee \underline{\neg v_8} \\
C_6 &= \underline{\neg v_4} \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \neg v_{16} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



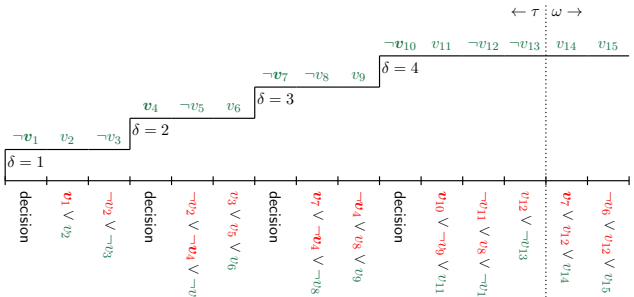
Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \underline{\neg v_4} \vee \underline{\neg v_8} \\
C_6 &= \underline{\neg v_4} \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \neg v_{16} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee v_{16}
\end{aligned}
$$



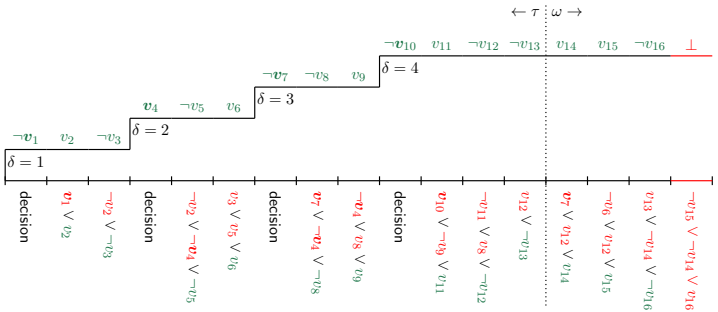
Watched Literals Example

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
C_4 &= v_3 \vee v_5 \vee v_6 \\
C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
C_6 &= \neg v_4 \vee v_8 \vee v_9 \\
C_7 &= v_{10} \vee \neg v_9 \vee v_{11} \\
C_8 &= \neg v_{11} \vee v_8 \vee \neg v_{12} \\
C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
C_{11} &= \neg v_6 \vee v_{12} \vee v_{15} \\
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
C_5 &= \underline{v_7} \vee \underline{\neg v_4} \vee \underline{\neg v_8} \\
C_6 &= \underline{\neg v_4} \vee \underline{v_8} \vee \underline{v_9} \\
C_7 &= \underline{v_{10}} \vee \underline{\neg v_9} \vee \underline{v_{11}} \\
C_8 &= \underline{\neg v_{11}} \vee \underline{v_8} \vee \underline{\neg v_{12}} \\
C_9 &= \underline{v_{12}} \vee \underline{\neg v_{13}} \\
C_{10} &= \underline{v_7} \vee \underline{v_{12}} \vee \underline{v_{14}} \\
C_{11} &= \underline{\neg v_6} \vee \underline{v_{12}} \vee \underline{v_{15}} \\
C_{12} &= \underline{v_{13}} \vee \underline{\neg v_{14}} \vee \underline{\neg v_{16}} \\
C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee \underline{v_{16}}
\end{aligned}
$$



Watched Literals Example

$$
\begin{aligned}
C_1 &= \underline{v_1} \vee \underline{v_2} \\
C_2 &= \underline{\neg v_2} \vee \underline{\neg v_3} \\
C_3 &= \underline{\neg v_2} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \\
C_4 &= \underline{v_3} \vee \underline{v_5} \vee \underline{v_6} \\
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C_{13} &= \underline{\neg v_{15}} \vee \underline{\neg v_{14}} \vee \underline{v_{16}}
\end{aligned}
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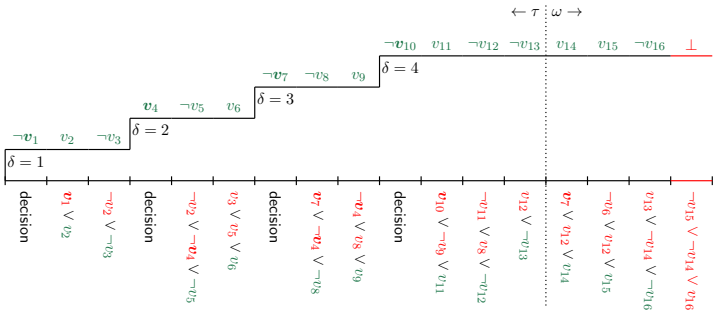

Watched Literals Example

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
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C_{14} &= \neg v_6 \vee v_7 \vee v_{12}
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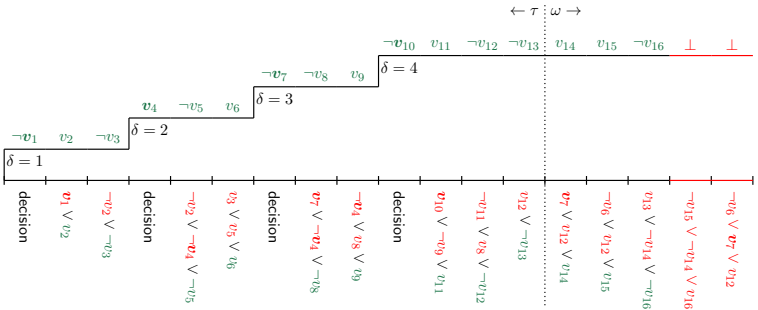

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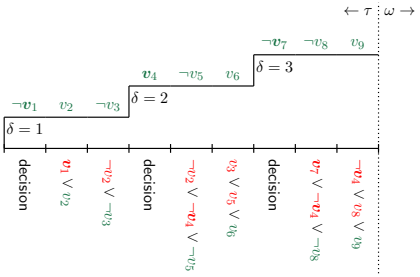

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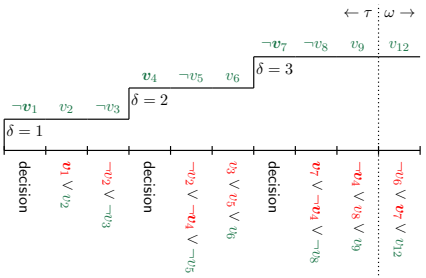

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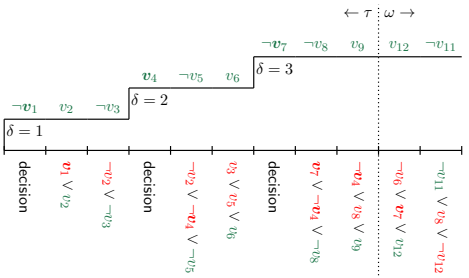

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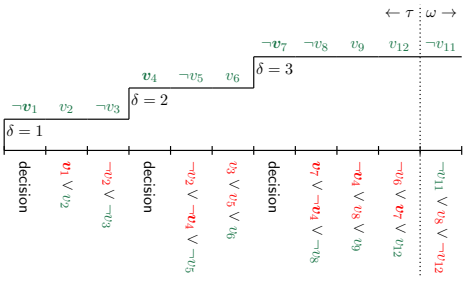

Properties

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Conflicting watch literals always in ω	

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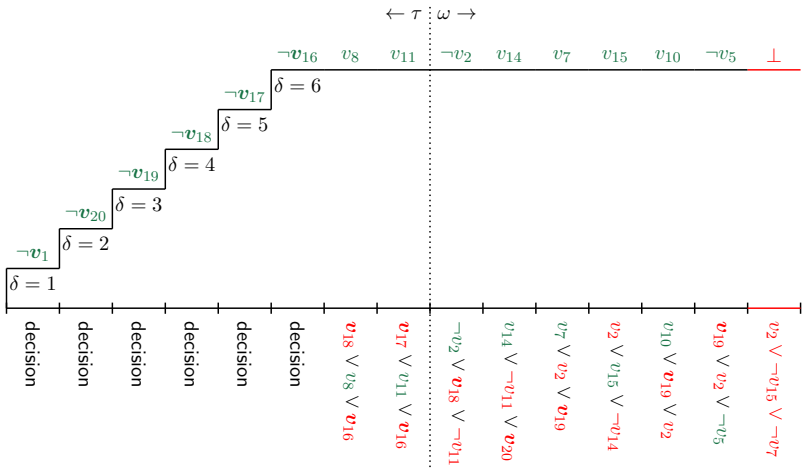
Conflicting watch literals always in ω	Yeah!
Conflicting watch literals in conflicts are at the same level	Nope
Backtracking unassigns conflicting watched literals	Nope
Conflict always at highest level	Nope
After backtracking, no clause can be propagating	Actually, they can...

Particular Case - One Literal at Highest Level [Möhle and Biere, 2019]



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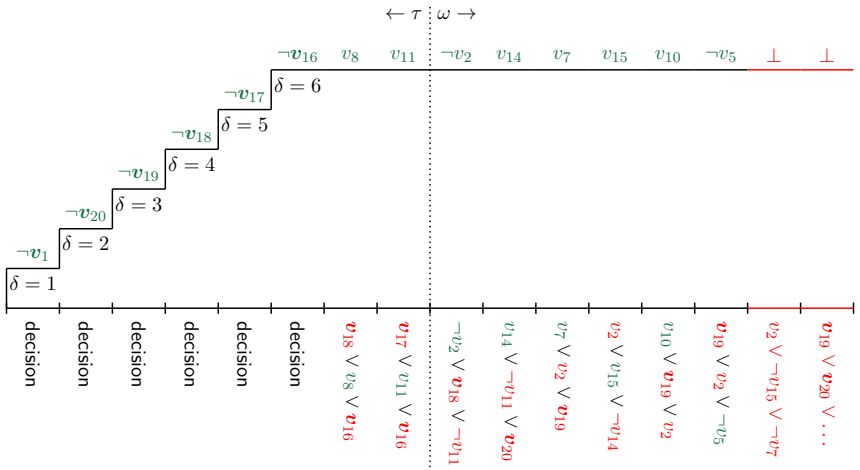

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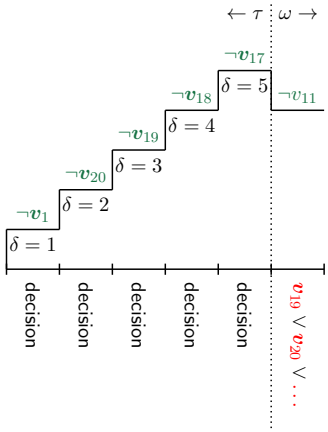

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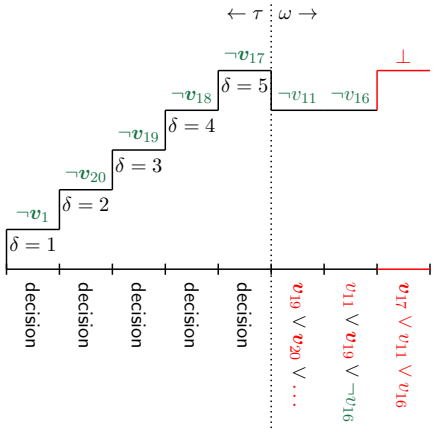

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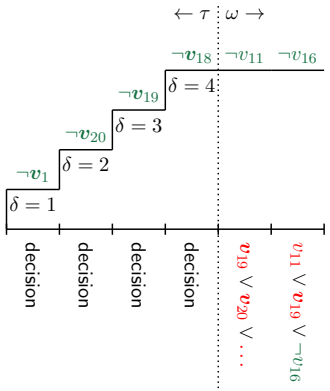

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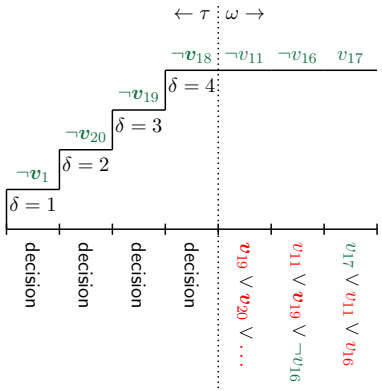
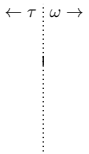


Feature - Blockers

Consider the clause $C = v_1 \vee v_2 \vee v_3 \vee v_4$ watched by v_1 and v_2 and the trail $\pi = \{v_3\}$. If $\neg v_1$ is propagated, Invariant "Watched literals" is violated. Should C change its watch literals ?

$$C = \underline{\quad v_1} \vee \underline{\quad v_2} \vee \quad v_3 \vee \quad v_4$$



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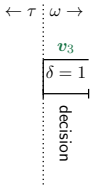

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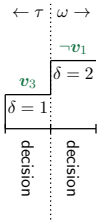

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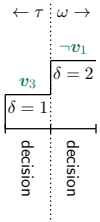

$C = \underline{v_1} \vee \underline{v_2} \vee \boxed{v_3} \vee v_4$

Invariant

(Blocked watch literals) Let $\pi = \tau \cup \omega$ be a partial assignment, and $C = \ell_1 \vee \ell_2 \vee \ldots$ be a clause watched by ℓ_1 and ℓ_2. If $\neg\ell_1 \in \tau \vee \neg\ell_2 \in \tau$, then C is satisfied.

Blockers: Problem in CB

$$C_1 = \underline{v_1} \vee \underline{v_2} \vee v_3$$
$$C_2 = \underline{\neg v_2} \vee \underline{v_4}$$
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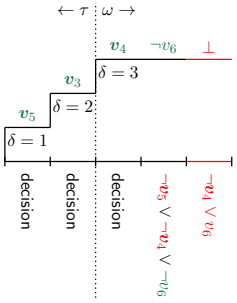

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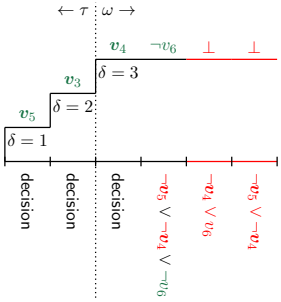
C_6 &= \underline{\neg v_5} \vee \underline{\neg v_4}
\end{aligned}
$$



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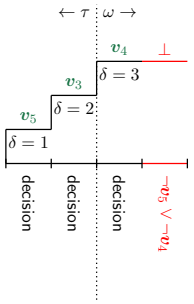

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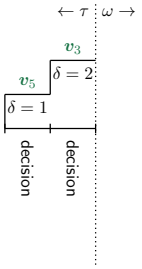

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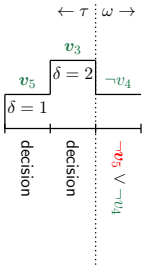

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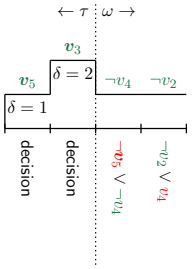

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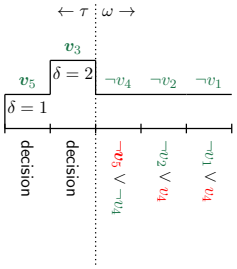

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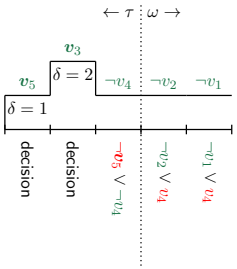

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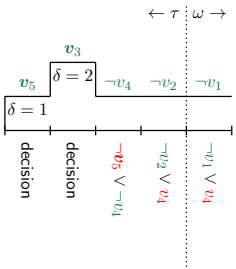

Blockers: Problem in CB

$$C_1 = \underline{\quad v_1} \vee \underline{\quad v_2} \vee \boxed{\quad v_3}$$
$$C_2 = \underline{\neg v_2} \vee \underline{\quad v_4}$$
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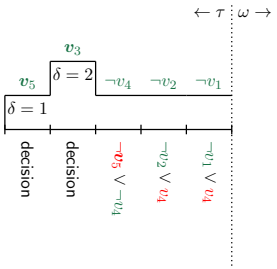

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\end{aligned}
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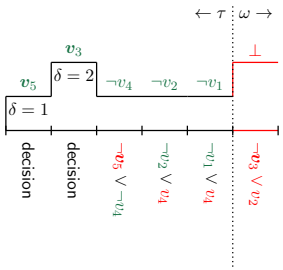

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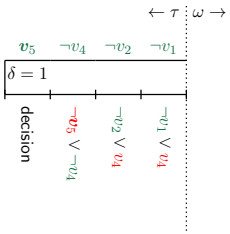

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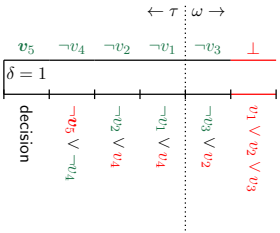

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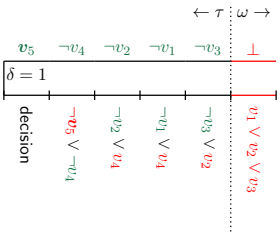

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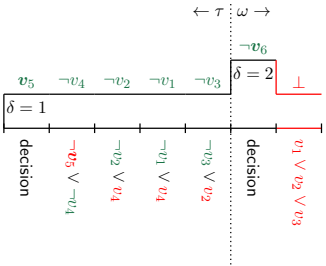

Weak Blockers [Coutelier, 2023]

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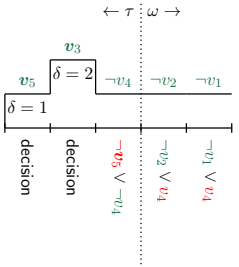

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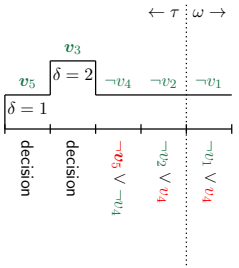

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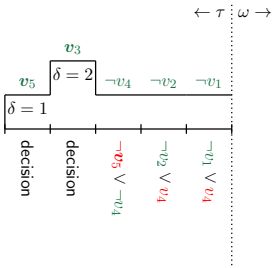

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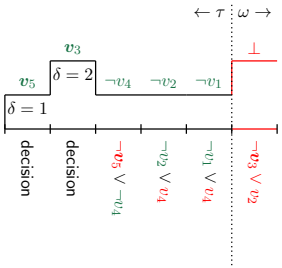

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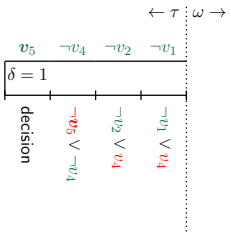

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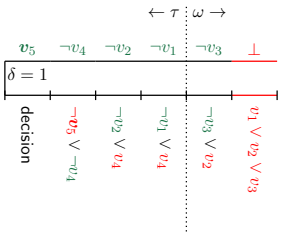

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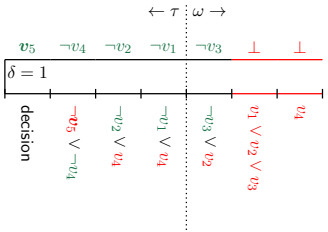

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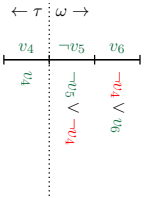
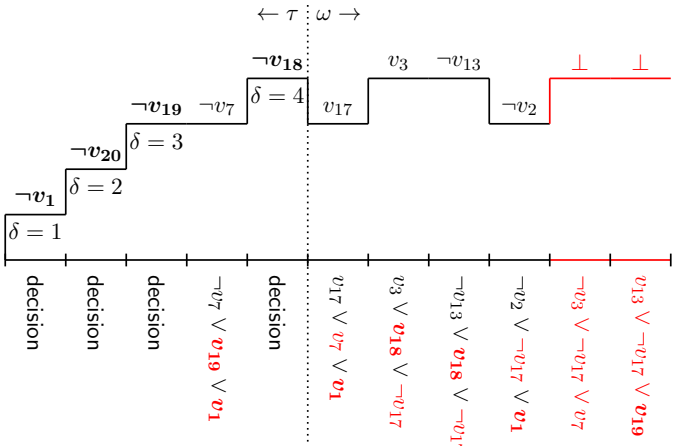


Strong Chronological Backtracking

In Weak Chronological Backtracking, it is possible to miss some implications. This is not desirable since it can lead to making decisions where a unit propagation would have been enough.
Strong Chronological Backtracking is a way to avoid this. It strengthens the invariants to avoid missing implications.

Multiple Conflicts at Max Level



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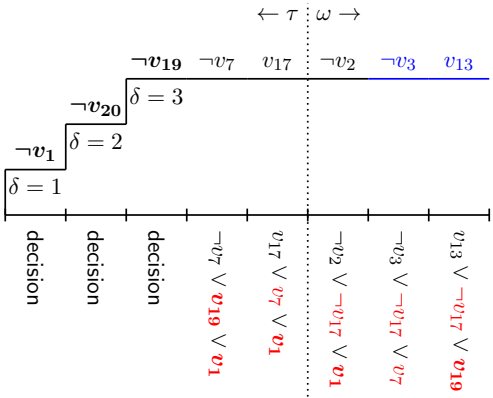

Missed Lower Implication [Nadel, 2022]

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A clause is unisat if it is satisfied by a single literal and all other literals are falsified.

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What is a missed lower implication?

In CB, it is possible that a clause C is satisfied by a literal ℓ at a level δ before it become unisat later with all falsified literals at a decision level lower than δ. The literal ℓ could have been propagated at a lower level. This is a missed lower implication.

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Why do we care?

When backtracking to level $\delta - 1$, C will become propagating, but it will not be detected. This can lead to making decisions where a unit propagation would have been enough.

Missed Lower Implication Example

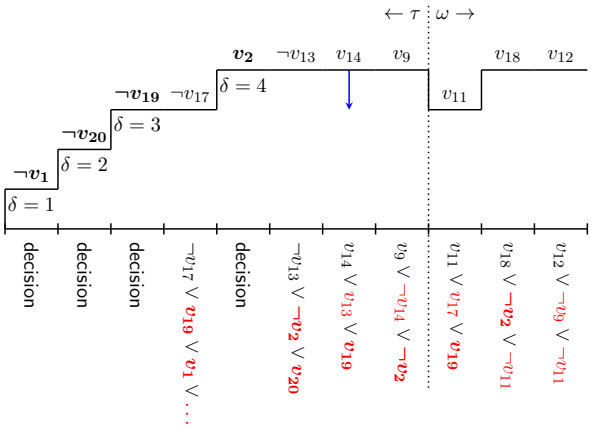

Figure: The literal v_{14} is a missed lower implication because the clause $v_{14} \vee \neg v_{11} \vee v_{20}$ became unit at decision level 3 when v_{11} was being propagated.

Re-implication

Detecting MLI

It is possible to detect missed lower implications while searching for a replacement literal in the clause. This is slightly worse than the original algorithm, but it is still linear.

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What to do with them?

We can store them in a re-implication list ρ and re-propagate them at the end of literal propagation.

Re-implication Difficulties

Chained re-implication

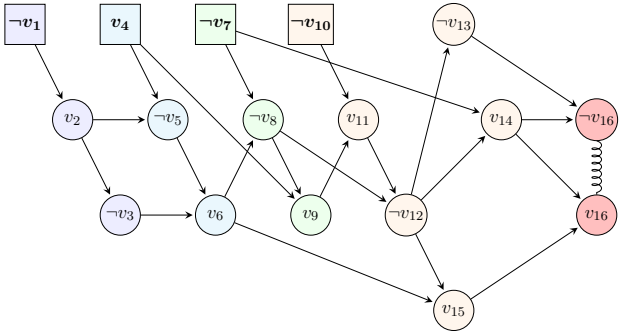
When re-implying a literal, it might itself cause a missed lower implication. Therefore, it is not sufficient to change the reason of the literal, we also need to re-propagate it. This can be expensive.

Level collapsing

It is possible that a decision is a missed lower implication. In this case, we need to collapse the entire level by 1. If the decision is re-implied lower than its current level minus one, then the level can only be collapse by one.

Implication Graph

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
C_4 &= v_3 \vee v_5 \vee v_6 \\
C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
C_6 &= \neg v_4 \vee v_8 \vee v_9 \\
C_7 &= v_{10} \vee \neg v_9 \vee v_{11} \\
C_8 &= \neg v_{11} \vee v_8 \vee \neg v_{12} \\
C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
C_{11} &= \neg v_6 \vee v_{12} \vee v_{15} \\
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



Topological Order

FUIP assumption

The FUIP algorithm assumes that the partial assignment π is a topological order of the implication graph.

Invariant

(Topological order) Let $\pi = \tau \cup \omega$ be a partial assignment. The implication graph of π is a DAG, and π is a topological order of the implication graph.

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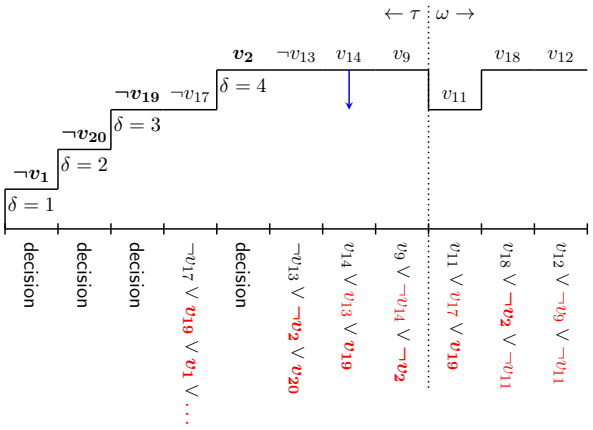
Reimplication breaks the topological order

Since the reason for the propagation of the literal ℓ changes, so does the implication graph. Furthermore, the literal ℓ is detected to be a missed lower implication after it was propagated. Therefore, at least one of the literals of the reason for the missed lower implication was propagated after ℓ. The topological order is broken.

Missed Lower Implication Example Reminder



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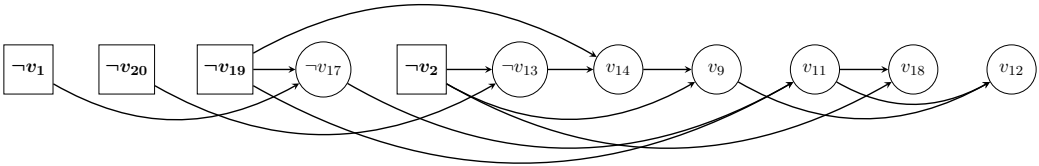
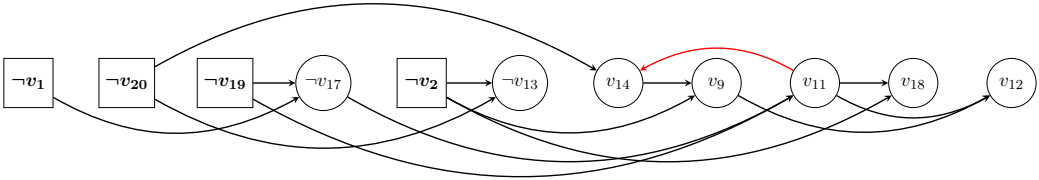
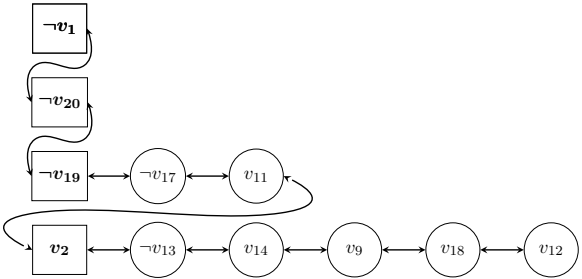


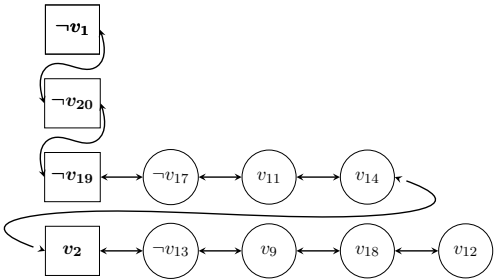
Broken Topological Order



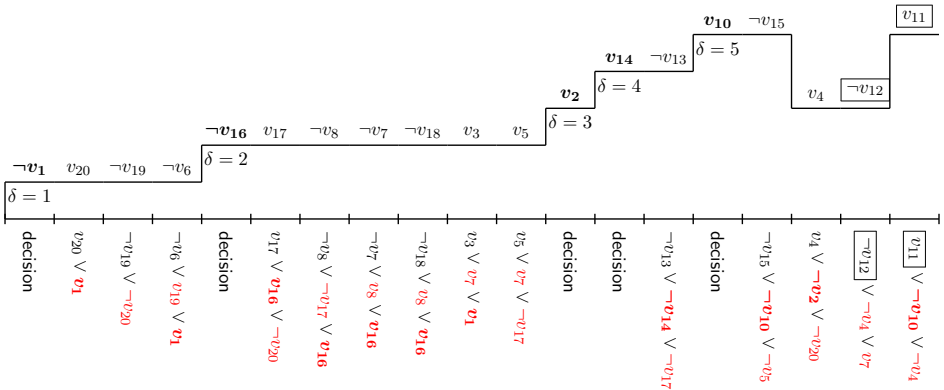
Solution 1 - Literal Linked List [Nadel, 2022]



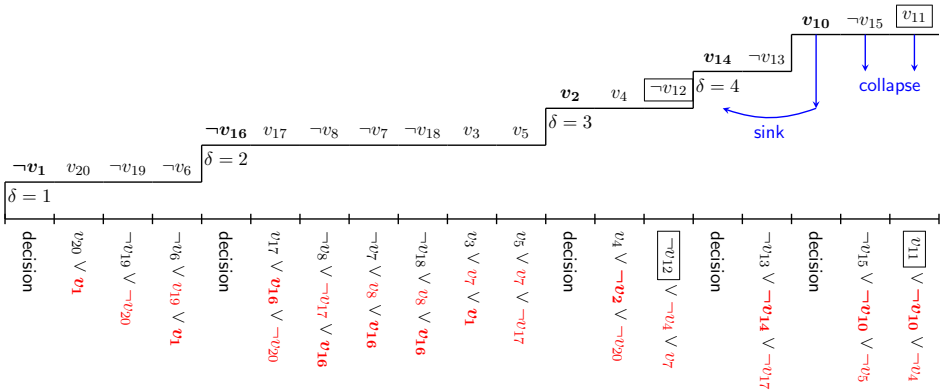
Solution 1 - Literal Linked List [Nadel, 2022]



Solution 2 - Trail Reordering and Literal Sinking [Coutelier, 2023]



Solution 2 - Trail Reordering and Literal Sinking [Coutelier, 2023]



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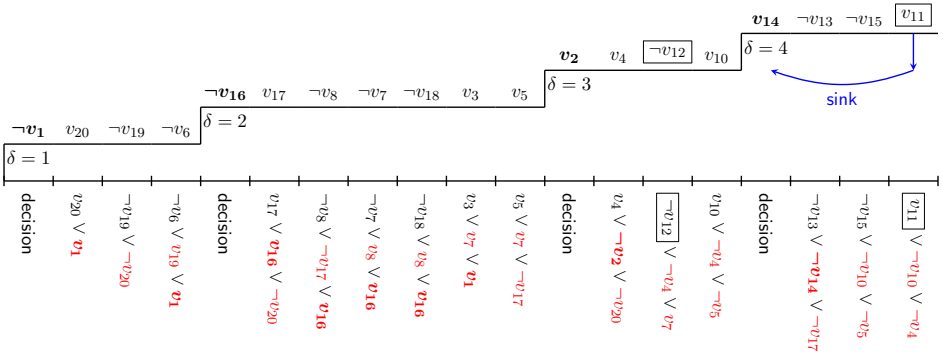

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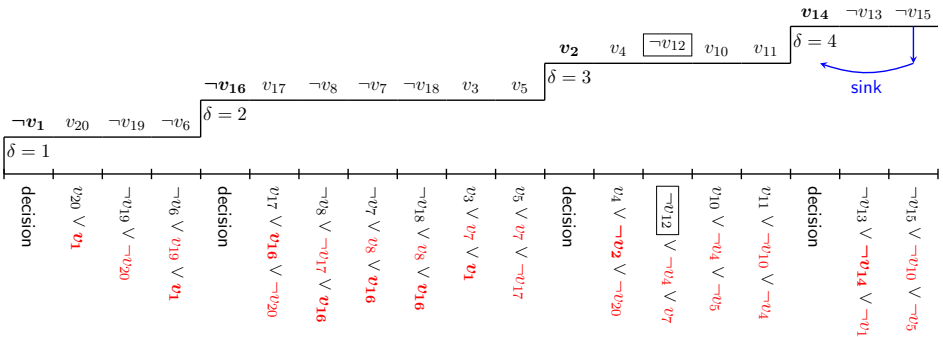

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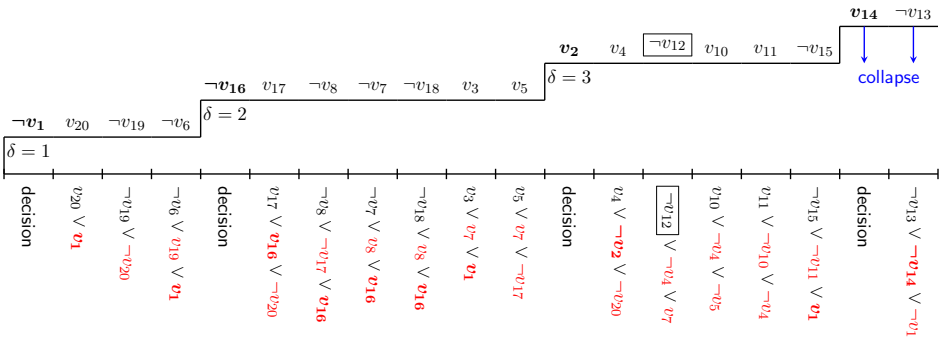

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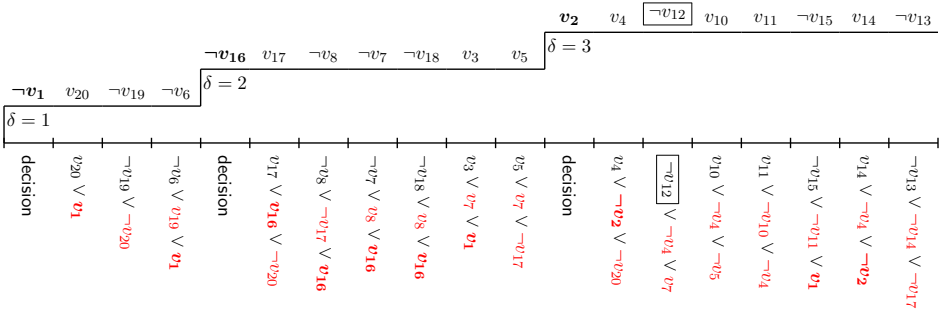
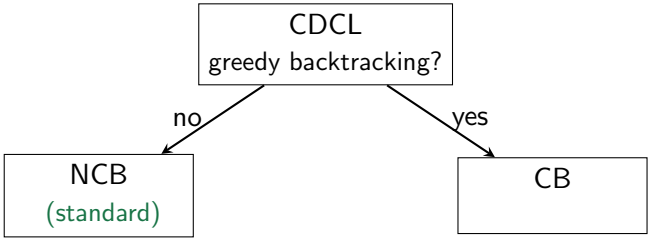


Backtracking Strategies

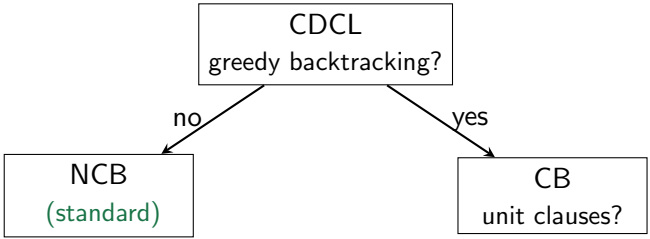
Backtracking Strategies

CDCL
greedy backtracking?

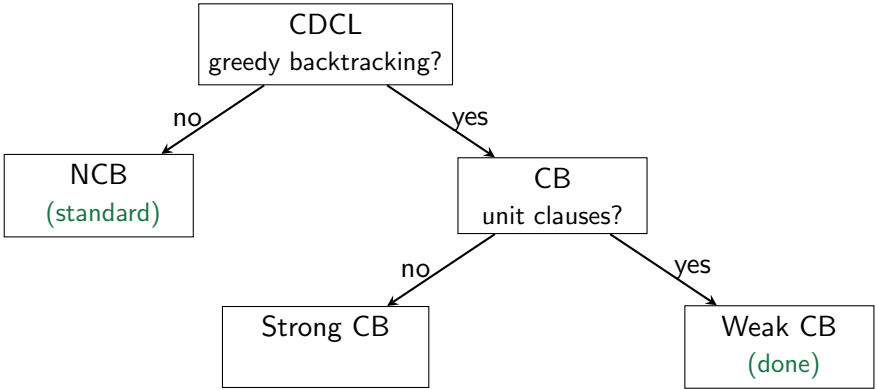
Backtracking Strategies



Backtracking Strategies



Backtracking Strategies



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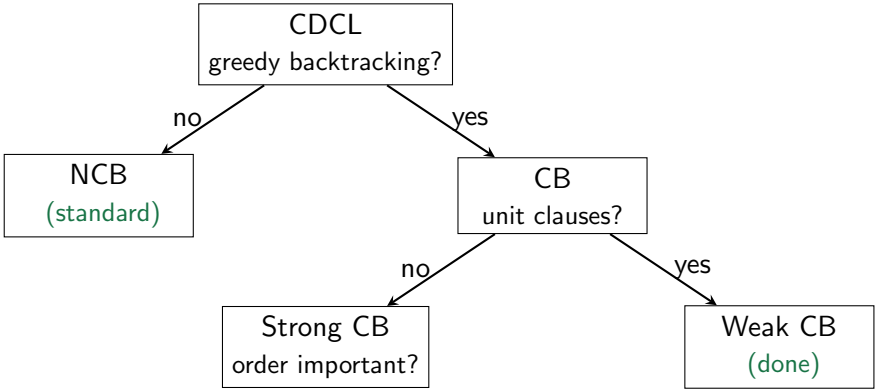
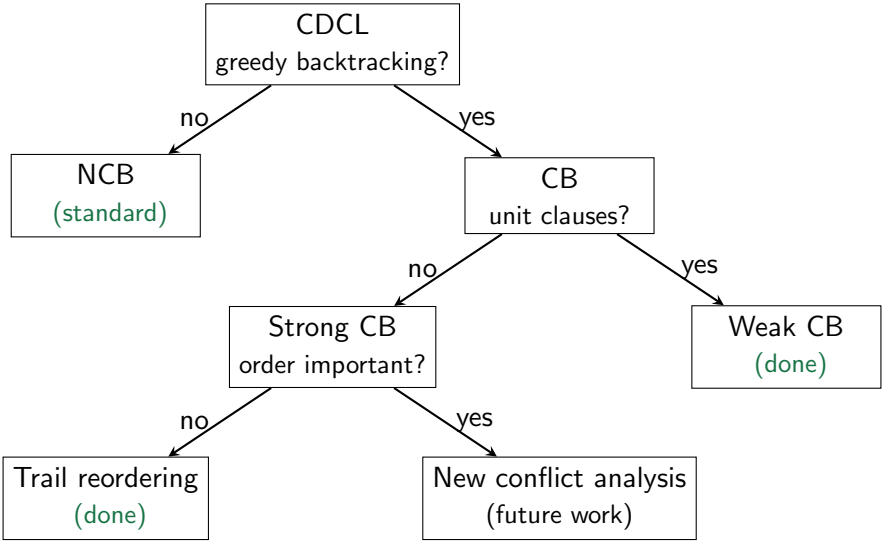


Backtracking Strategies



Future Work

- Support for Strong Chronological Backtracking in the SAT solver of `modulariT`.
- Lazy re-implication for Strong Chronological Backtracking.
- Integration in SMT solver `modulariT`.
- Less aggressive watch literal changes.
- Polishing the debugging tool: Invariant selection, better visualization, execution comparison, execution saving and replaying, etc.

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Problem - No Backtracking Guarantee

$$C_1 = \underline{\neg v_1} \vee \underline{\quad v_2}$$
$$C_2 = \underline{\neg v_1} \vee \underline{\neg v_3} \vee \quad v_4$$
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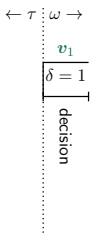

Problem - No Backtracking Guarantee

$$
\begin{aligned}
C_1 &= \underline{\neg v_1} \vee \underline{\ v_2} \\
C_2 &= \underline{\neg v_1} \vee \underline{\neg v_3} \vee \underline{\ v_4} \\
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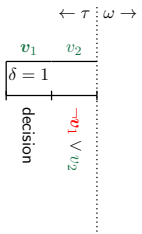
C_6 &= \underline{\neg v_3} \vee \underline{\neg v_4} \vee \neg v_5 \vee \neg v_6
\end{aligned}
$$



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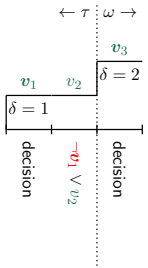

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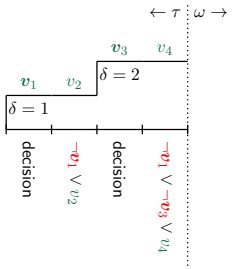

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$$C_1 = \underline{\neg v_1} \vee \underline{\ v_2}$$
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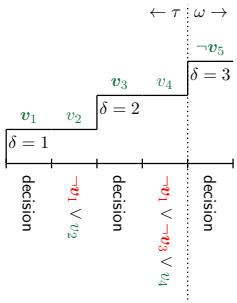

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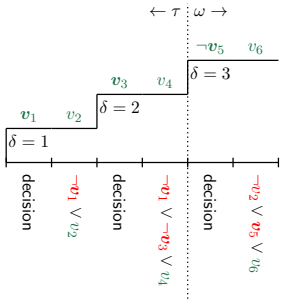

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$$
\begin{aligned}
C_1 &= \underline{\neg v_1} \vee \underline{\ v_2} \\
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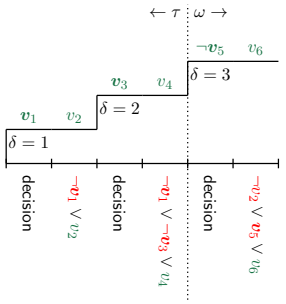
C_6 &= \underline{\neg v_3} \vee \underline{\neg v_4} \vee \underline{\neg v_5} \vee \underline{\neg v_6} \\
C_7 &= \underline{\neg v_1} \vee \underline{\ v_5}
\end{aligned}
$$



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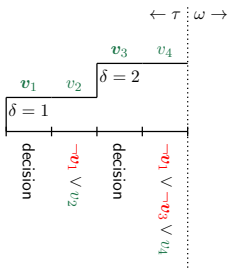

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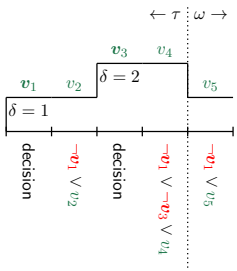

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$$C_1 = \underline{\neg v_1} \vee \underline{\ v_2}$$
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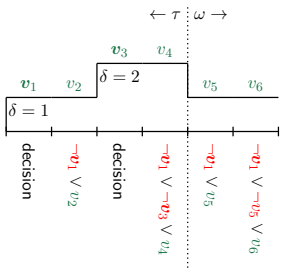

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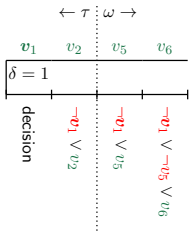

Solution - Change Watched Literals

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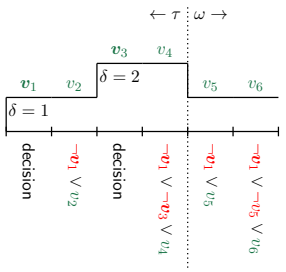

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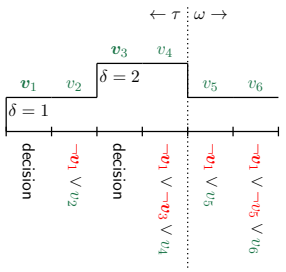

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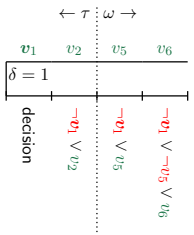
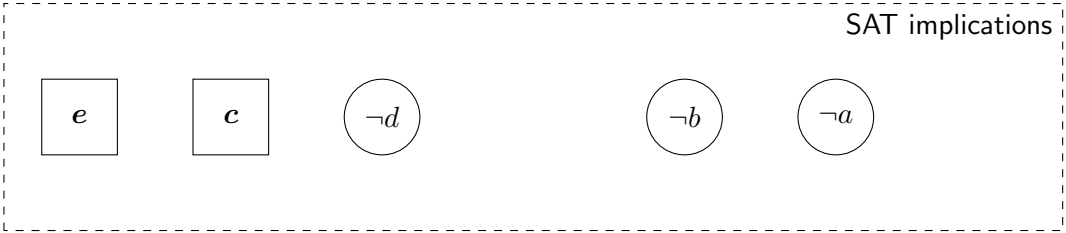
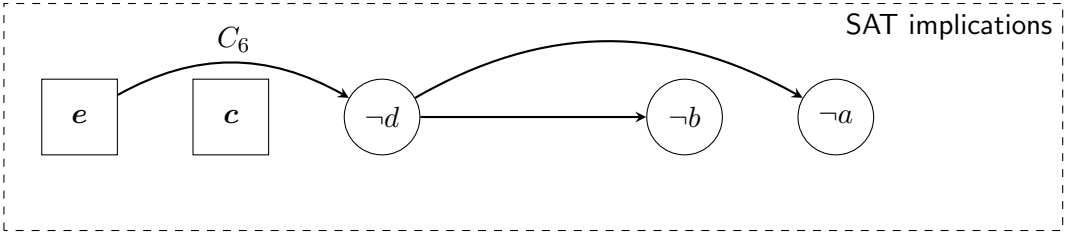
$$C_7 = \underline{\neg v_1} \vee \underline{\ v_5}$$
$$C_8 = \underline{\neg v_6} \vee \neg v_5 \vee \neg v_1 \vee \underline{\neg v_3}$$



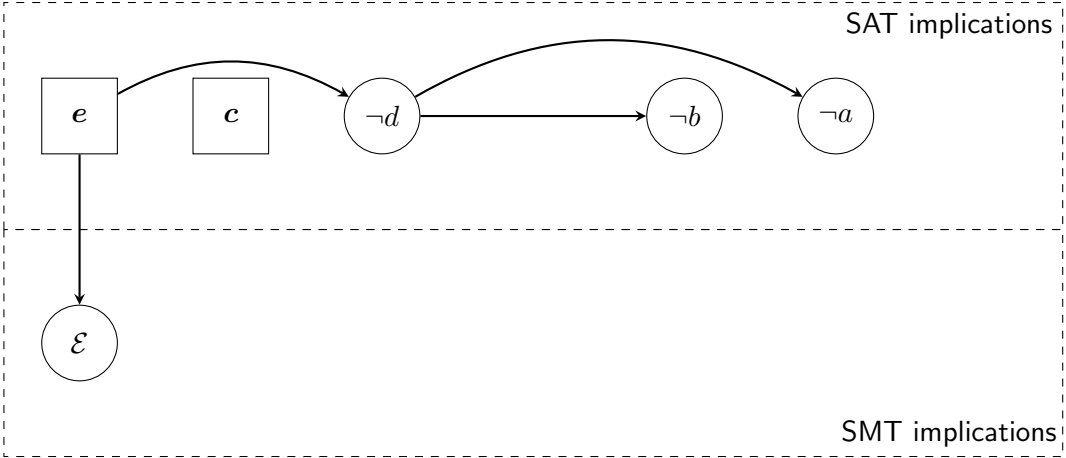
SAT and SMT Trails



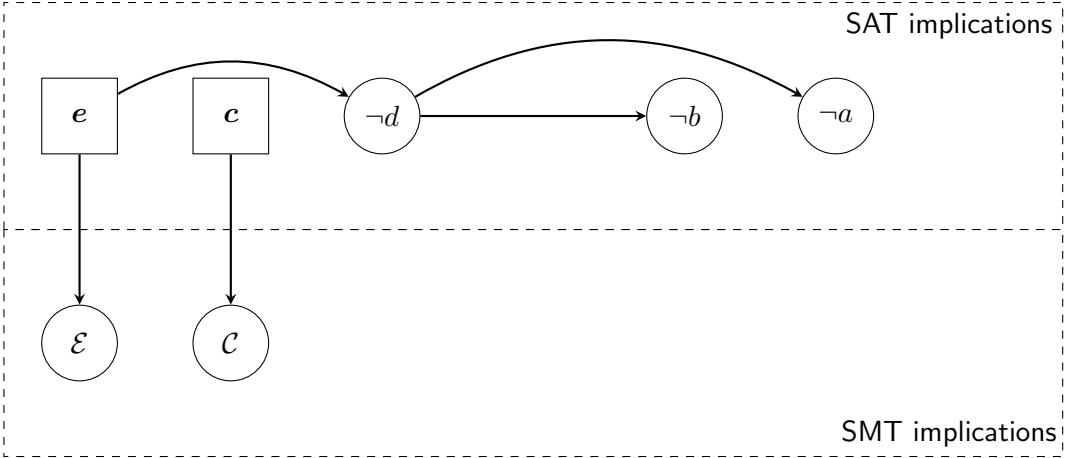
SAT and SMT Trails



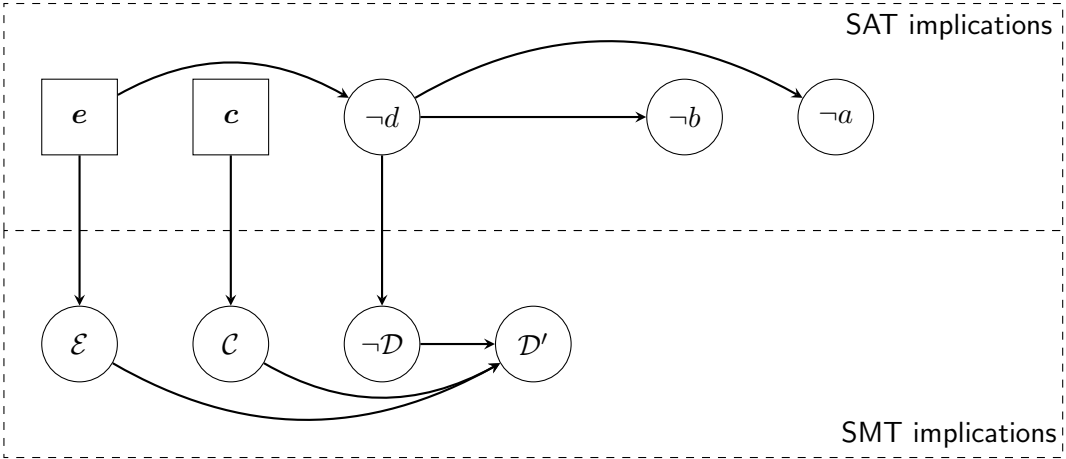
SAT and SMT Trails



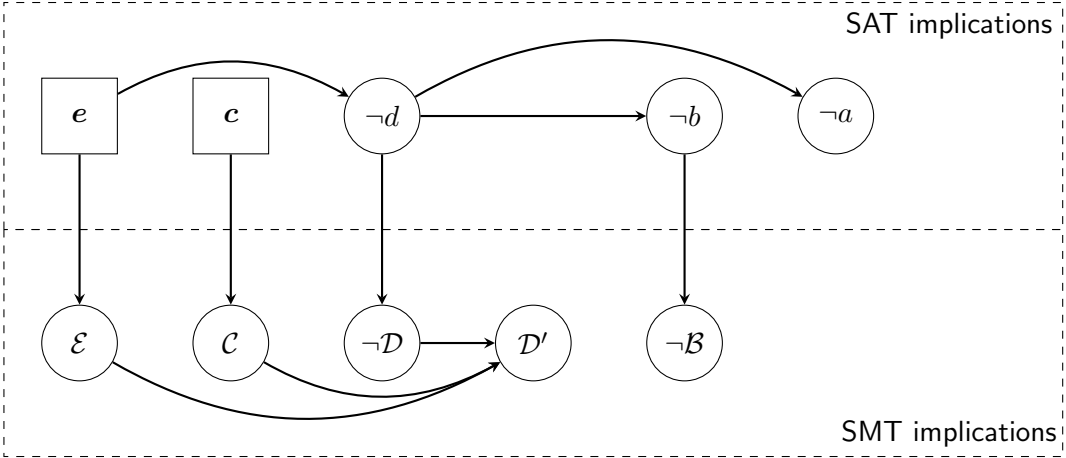
SAT and SMT Trails



SAT and SMT Trails



SAT and SMT Trails



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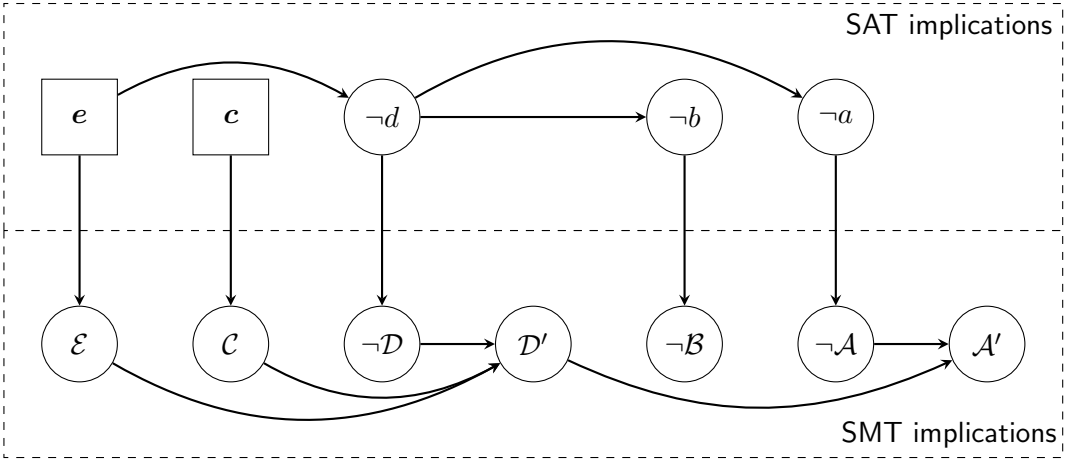

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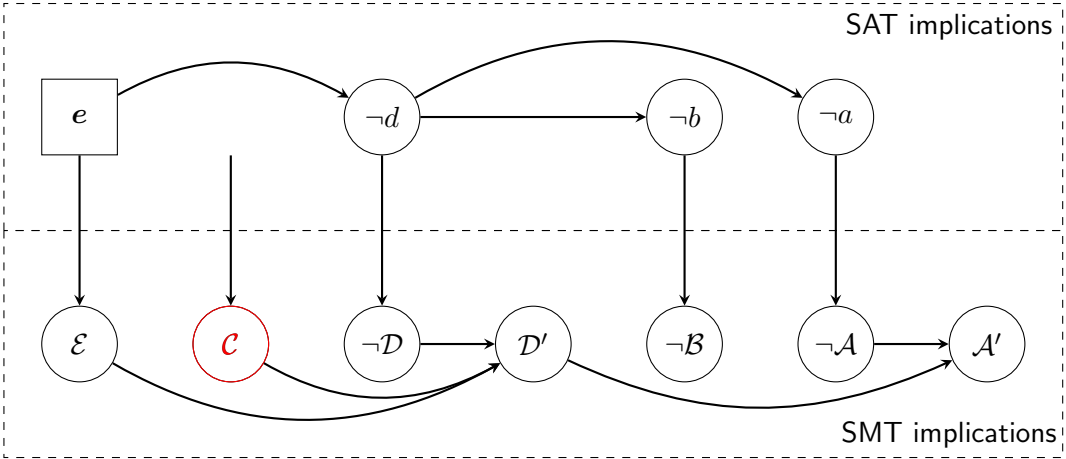
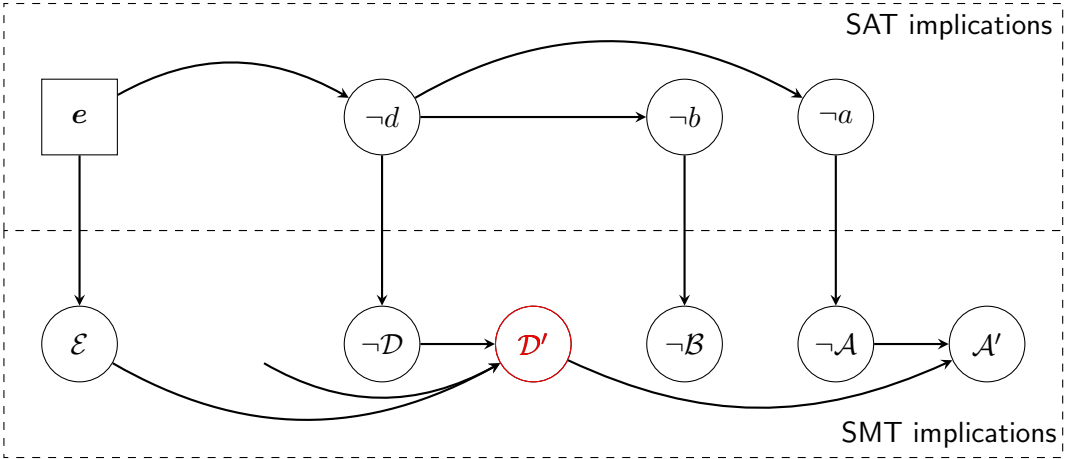
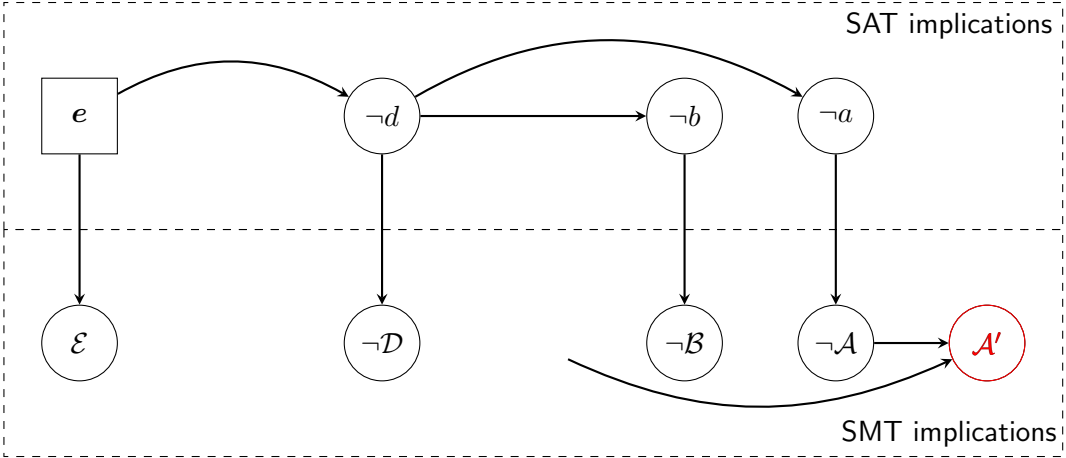


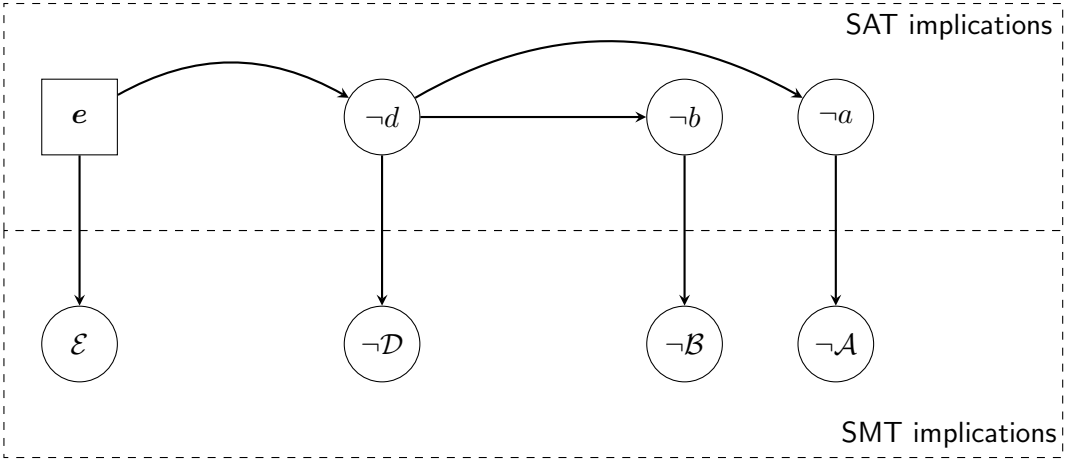
SAT and SMT Trails



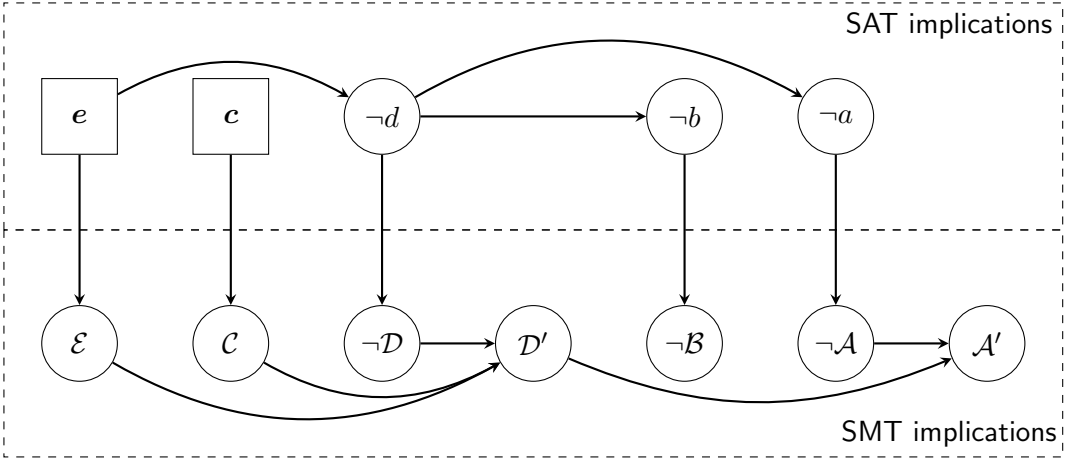
SAT and SMT Trails



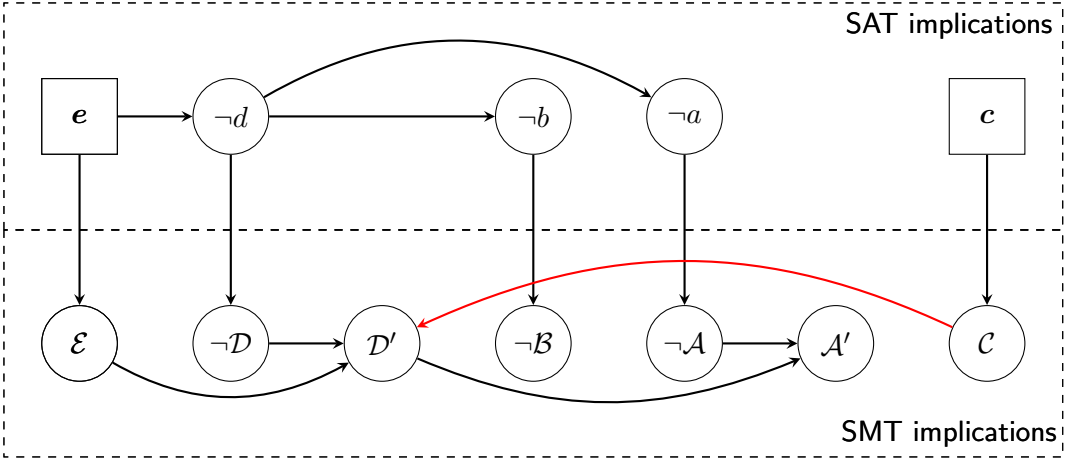
SAT and SMT Trails



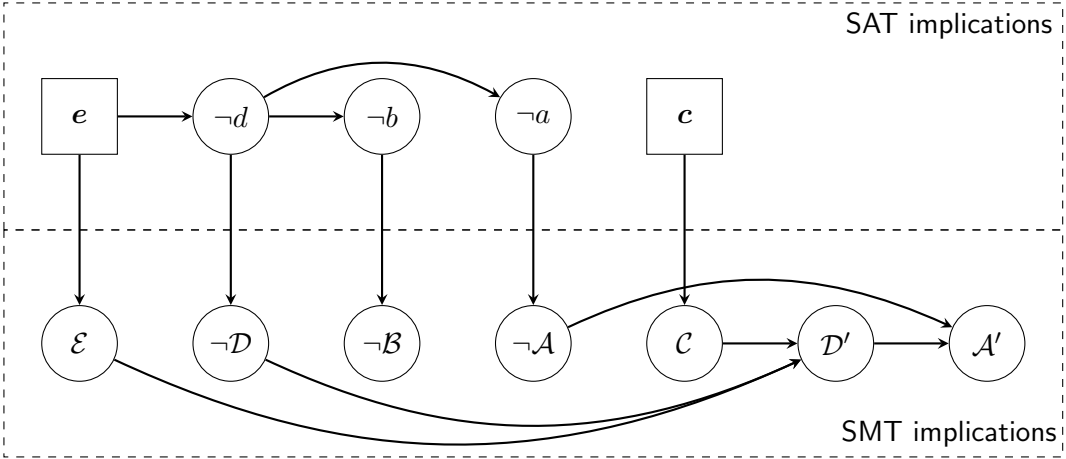
Moving Literals



Moving Literals

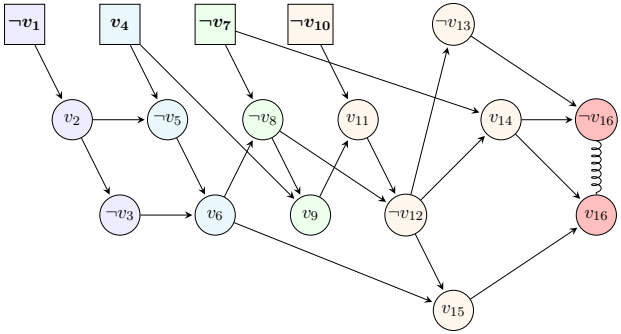


Moving Literals



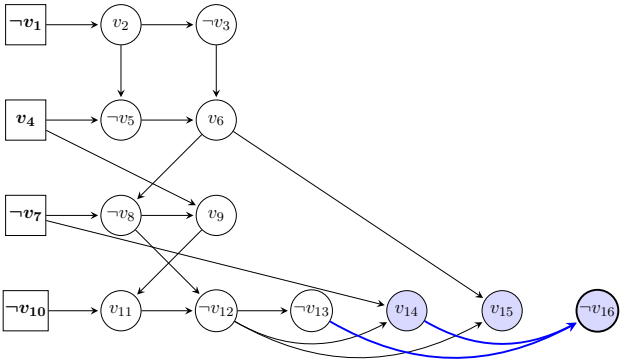
Conflict Analysis

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
C_4 &= v_3 \vee v_5 \vee v_6 \\
C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
C_6 &= \neg v_4 \vee v_8 \vee v_9 \\
C_7 &= v_{10} \vee \neg v_9 \vee v_{11} \\
C_8 &= \neg v_{11} \vee v_8 \vee \neg v_{12} \\
C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
C_{11} &= \neg v_6 \vee v_{12} \vee v_{15} \\
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



Conflict Analysis

$$
\begin{aligned}
C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
C_4 &= v_3 \vee v_5 \vee v_6 \\
C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
C_6 &= \neg v_4 \vee v_8 \vee v_9 \\
C_7 &= v_{10} \vee \neg v_9 \vee v_{11} \\
C_8 &= \neg v_{11} \vee v_8 \vee \neg v_{12} \\
C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
C_{11} &= \neg v_6 \vee v_{12} \vee v_{15} \\
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



Conflict Analysis

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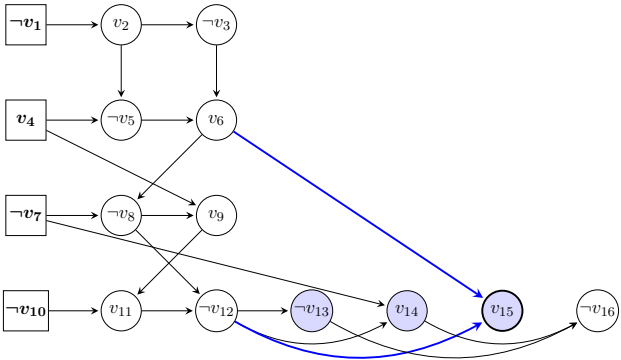
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



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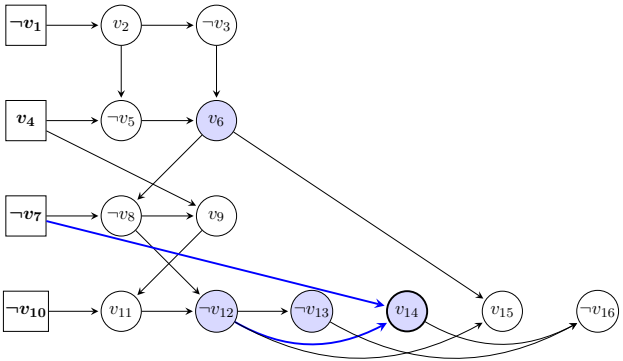
C_{12} &= v_{13} \vee \neg v_{14} \vee \neg v_{16} \\
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



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C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
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C_5 &= v_7 \vee \neg v_4 \vee \neg v_8 \\
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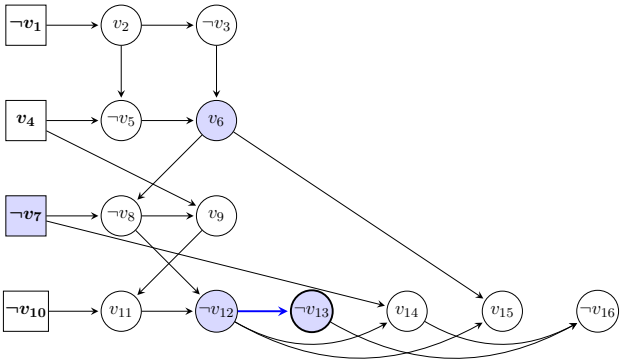
C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$



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C_1 &= v_1 \vee v_2 \\
C_2 &= \neg v_2 \vee \neg v_3 \\
C_3 &= \neg v_2 \vee \neg v_4 \vee \neg v_5 \\
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C_9 &= v_{12} \vee \neg v_{13} \\
C_{10} &= v_7 \vee v_{12} \vee v_{14} \\
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C_{13} &= \neg v_{15} \vee \neg v_{14} \vee v_{16}
\end{aligned}
$$